

White dwarf cooling and large extra dimensions

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Abstract

Theories of fundamental interactions with large extra dimensions have recently become very popular. Astrophysical bounds from the Sun, red-giants and SN1987a have already been derived by other authors for the theory proposed by Arkani-Hamed, Dimopoulos and Dvali. In this paper we consider G117-B15A pulsating white dwarf (ZZ Ceti star) for which the secular rate at which the period of its fundamental mode increases has been accurately measured and claimed that this mode of G117-B15A is perhaps the most stable oscillation ever recorded in the optical band. Because an additional channel of energy loss (Kaluza-Klein gravitons) would speed up the cooling rate, one is able to use the aforementioned stability to derive a bound on theories with large extra dimensions. Within the framework of the theory with large extra dimensions proposed by Arkani-Hamed, Dimopoulos and Dvali we find the lower bound on string compactification scale $M_s > 14.3 \text{ TeV}/c^2$ which is more stringent than solar or red-giant bounds.

1 Introduction

The interest in physical theories with extra spatial dimensions has recently experienced considerable revival. Multidimensional theories with compact spatial dimensions having inverse radius at the order of the GUT scale were investigated intensively in the 80-ties [1]. Later developments in string theory initially due to Anoniadis [2] later supported by [3] suggested that it could be possible to have a much lower string or compactification scale. In particular, it has been conjectured [4] that compactification scale could be at the order of a TeV, corresponding to a weak-scale string theory. Such a low string scale is attractive from experimental perspective since the string state spectrum would now become accessible. In this class of theories, gravity is essentially $n + 4$ dimensional whereas all other physical fields are confined to 4-dimensional brane. The relation between the Planck mass in 4 dimensions ($M_{Pl} = 1.2 \cdot 10^{19} \text{ GeV}/c^2$) and the string mass scale in $4 + n$ dimensions M_s and the radius R of extradimensional space reads:

$$R^n = \left(\frac{\hbar}{c}\right)^n \frac{M_{Pl}^2}{M_s^{n+2}\Omega_n} \quad (1)$$

where Ω_n is the volume of the unit n -sphere. Present laboratory limits [5] give 1TeV as lower bound on M_s . Assuming that M_s is of order of TeV (sustaining the hopes of experimental verification of multidimensionality of the world) one can immediately rule out $n = 1$ because in that case one would expect modified gravity at distances $R \approx 10^{15} \text{ cm}$ which is not observed.

Last century witnessed a successful application of standard physics in elucidating the properties of celestial bodies which consequently can be used as a source of - sometimes strong - bounds. The idea that astrophysical considerations can constrain “exotic” physics is not a new one. For example it has been implemented to constrain the axion mass [6]. The main idea here is that if “exotic” physics to be tested predicts the existence of weakly interacting particles which can be produced in stellar interiors such weakly interacting particles would serve as an additional source of energy loss in many ways influencing the course of stellar evolution. In the context of multidimensional physics excitation of Kaluza-Klein gravitons play the role of stellar extra coolant.

There are three main sources of astrophysical bounds invoked in this context. First is the Sun which is a hydrogen burning main sequence star with radiative interior. Back reaction in response to increased energy loss results in rising the internal temperature and shrinking of the radiative interior (for details see [6]). Because enhanced temperature increases the rates of nuclear reactions this would exacerbate solar neutrino problem. Moreover helioseismology provides an accurate estimate of the internal profile of isothermal sound speed squared [7]. The most accurate value of this quantity, which is pro-

portional to temperature, refers to the radius $r = 0.2 R_{\odot}$. This provides an effective mean to constrain exotic sources of energy loss.

The second source of bounds comes from red giants - stellar evolutionary phase after the hydrogen has been exhausted and which lasts until the helium ignition takes place during the so called helium flash. Additional cooling would results in one of the following effects. First of all helium flash would not occur if cooling was too effective. If additional cooling is less effective (so that helium flash eventually takes place) the red giant tip on the Hertzsprung-Russel diagram would be located higher above the Horizontal Branch and the time spent on a Horizontal Branch would be shortened. All these effects could be tested on HR diagrams for globular clusters [6].

The last and the most effective of traditional sources of bounds comes from SN1987a - more specifically from the observed duration of neutrino pulse. The pulse would be shortened had the nascent neutron star cooled down more rapidly than standard theory predicted. These considerations have been applied, in the context of large extra-dimensions in works of Barger et al. [11], Cullen and Perelstein [12] and Cassisi et al [14] (see also [13]).

In this paper we consider another class of astrophysical objects for which the cooling rate is known from observations, namely the pulsating white dwarfs. We will focus our attention on ZZ Ceti star G117-B15A which was considered previously as a tool of testing fundamental physics (in the context of axion emission) [8, 6].

2 White dwarf cooling in the presence of large extra dimensions

White dwarfs such like G117-B15A are final stages of evolution of stars with masses smaller than $8 M_{\odot}$. They have inner degenerate core composed of carbon and oxygen which is an internal energy reservoir and a thin non-degenerate envelope made up of He and H shells. Having exhausted their nuclear fuel these stars can only contract and cool down. Some of them e.g. ZZ Ceti stars are pulsating and their pulsation period increases as the star cools down [9, 10]. The rate of period increase \dot{P}/P is proportional to temperature decrease rate \dot{T}/T .

At an energy scale much lower than the string scale, one can construct an effective theory of KK gravitons interacting with the standard model fields [15]. Barger et al. [11] have calculated Kaluza-Klein graviton emissivities in five processes interesting from astrophysical perspective: photon-photon annihilation, electron - positron annihilation, Gravi-Compton-Primakoff scattering, Gravi-bremsstrahlung in a static electric field and nucleon-nucleon bremsstrahlung. Kaluza-Klein gravitons can couple to photons which leads to the first process,

similarly electron-positron pair can annihilate into Kaluza-Klein gravitons, in the third process scattering of photons by electrons may lead to Kaluza-Klein graviton emission via Compton or Primakoff processes (conversion of photons into Kaluza-Klein gravitons) - respective Feynman diagrams can be found in [11]. First of the last two processes consists in bremsstrahlung emission of Kaluza-Klein gravitons in static electric field of ions, the next one is similar - the bremsstrahlung emission of gravitons by nucleons (in electric field of nucleons). One can expect that first process becomes effective in hot stars where the photon density is large enough, the second one in stars with abundant electron-positron pairs (i.e. mainly the protoneutron or young neutron stars) etc.

Because white dwarfs are dense and cool one can expect that dominant process of Kaluza-Klein graviton emission is gravi-bremsstrahlung of electrons. The specific (mass) emissivity estimated by Barger et al. [11] is

$$\epsilon = 5.86 \cdot 10^{-75} \frac{T^3 n_e}{\rho M_s^4} \sum_j n_j Z_j^2 \quad \text{for } n = 2 \quad (2)$$

$$\epsilon = 9.74 \cdot 10^{-91} \frac{T^4 n_e}{\rho M_s^5} \sum_j n_j Z_j^2 \quad \text{for } n = 3 \quad (3)$$

where: T is the temperature of isothermal core, ρ is the density, n_e and n_j are the number densities of electrons and ions respectively. Total Kaluza-Klein graviton luminosity can be obtained as

$$L_{KK} = \int_0^{M_{WD}} \epsilon \, dm \quad (4)$$

Extensive theoretical studies of non-radial oscillations of white dwarfs had been carried out long time ago [9]. Appropriate theory of stellar oscillations consists in linearising the Poisson equation as well as equations of momentum, energy and mass conservation with respect to small non-radial perturbations. These perturbations (which in ZZ Ceti stars are the so called g-modes) can have oscillatory behavior $\propto \exp(-i\sigma t)$ where $\sigma^2 \propto -A$. The quantity A (which also illustrates the connection between oscillations and convective instability) is determined by thermodynamical properties of stellar matter: $A = \frac{d \ln \rho}{dr} - \frac{1}{\Gamma_1} \frac{d \ln p}{dr}$ where: ρ denotes density, r - radial coordinate, p is the pressure and Γ_1 is the adiabatic index [16]. Now, for a zero-temperature degenerate electron gas $A = 0$ meaning that no g-modes are supported. However, if non-zero thermal effects are taken into account one can show [9] that $A \propto T^2$ and consequently $\frac{1}{P} \propto T$ i.e. the periods scale like $1/T$ where T is the core temperature. Consequently, the increase of the pulsation period can be calculated from the following formula:

$$\frac{\dot{P}}{P} \propto -\frac{\dot{T}}{T} = -\frac{L}{c_v M_{WD} T} \quad (5)$$

where L is the total luminosity, c_v - average heat capacity and M_{WD} is the mass of the white dwarf. In the second part of the above formula (equality) the famous Mestel cooling law has been applied [17]. As mentioned above we assume that anomalous cooling rate observed in the form of pulsation period increase is due to Kaluza-Klein graviton emission. In such case we can write [8]

$$\frac{L_{KK}}{L_\gamma} = \frac{\dot{P}_{obs}}{\dot{P}_0} - 1 \quad (6)$$

where L_γ represents standard photon cooling, \dot{P}_{obs} represents the observed speed of pulsation period increase, \dot{P}_0 is an analogous quantity but without Kaluza-Klein cooling (theoretical).

Since its discovery in 1976 [18] G117-B15A has been extensively studied. Regarding its variability the observed periods are 215.2, 271 and 304.4 s together with higher harmonics and linear combinations thereof [19]. First estimates of the rate of increase of main pulsation period $P = 215.2$ s motivated Isern et al. [8] to discuss the effect of axion emission from this star. Very recently, with a much longer time interval of acquired data, Kepler et al. [21] recalculated the rate of period increase and found significantly lower value of $\dot{P} = (2.3 \pm 1.4) \times 10^{-15} \text{ s s}^{-1}$. Hence it has been claimed that the 215.2 s mode of G117-B15A is perhaps the most stable oscillation ever recorded in the optical band (with a stability compared to millisecond pulsars [22]).

This circumstance makes it possible to derive a bound on string mass scale M_s . Previous upper limits on \dot{P} would only allow to test whether such “peculiar” behavior (if true) could be a manifestation of large extra dimensions and whether it was consistent with other astrophysical bounds.

The white dwarf pulsator G117-B15A has a mass of $0.59 M_\odot$, effective temperature $T_{eff} = 11620 \text{ K}$ [23] and luminosity $\log(L/L_\odot) = -2.8$ [24] (i.e. $L_\gamma = 6.18 \times 10^{30} \text{ erg s}^{-1}$). Typical model for such CO star predicts the central temperature $T = 1.2 \times 10^7 \text{ K}$ [22] and matter density $\rho = 0.97 \times 10^6 \text{ g cm}^{-3}$. We assume that mean molecular weight per electron is $\mu_e \approx 2$. This allows us to estimate electron number density n_e . Following detailed calculations by Salaris [25] chemical composition of the white dwarf core has been taken as 83% by mass of oxygen and remaining 17% of carbon. Then by virtue of electric neutrality of the star one is able to estimate n_j . Now we can estimate the power radiated away from the white dwarf core in the form of Kaluza-Klein gravitons L_{KK} . In order to derive the lower bound on M_s one can take (in the role of \dot{P}_{obs} in (6)) an upper 2σ limit of \dot{P} equal to $5.1 \times 10^{-15} \text{ s s}^{-1}$ [21]. Following [22] we assume $\dot{P}_0 = 3.9 \times 10^{-15} \text{ s s}^{-1}$. This means that recently established secular stability of the fundamental oscillation mode of G117-B15A implies $L_{KK} < 0.308 L_\gamma$ which translates to:

$$M_s > 14.3 \text{ TeV}/c^2 \text{ for } n = 2$$

Respective graviton emission rates for $n = 3$ (and greater) theories turn out to be negligible, hence we do not quote the resulting numbers.

Our bound has been derived within a very simple law relating the observed rate of the pulsational period and the rate of the change of the period given by models when Kaluza-Klein gravitons are considered - essentially that proposed by Isern et al. [8]. The most self-consistent approach would be to run a set of evolutionary models of white-dwarfs and compare their predictions concerning \dot{P} with the observed value \dot{P}_{obs} . In the case of axion emission this has been done by Corsico et al. in [22] and the result obtained that way turned out to be in a very good agreement with the one obtained by applying the above mentioned much simpler method. The reason for this could be understood by very simple argument that the axion (or Kaluza-Klein gravitons as in our case) emissivity is dominated by the bremsstrahlung process in isothermal degenerate core. Based on the evolutionary white dwarf models the authors of [22] pointed out to a very important circumstance that additional cooling from the "exotic" physics affects the temperature profile in the innermost degenerate parts of the star. The structure of the outer (partially degenerate) layers depends on the temperature profile. However this profile is to great extent determined by the effective temperature (fixed by observations). In consequence the periods of oscillations (which depend on the structure of the star) remain almost unchanged when additional cooling (which in any case is a small correction to the energy budget) is present. On the other hand the value of \dot{P} is sensitive to additional cooling. This justifies the assumption that one can first identify the structure of the fiducial model (taken by us as that cited in [22]) without Kaluza-Klein graviton emission and then incorporate graviton emission in discussing the rate of secular changes of the period.

3 Conclusions

It is interesting to compare our result with existing astrophysical constraints on the string compactification scale within the framework of the theory proposed by Arkani-Hamed, Dimopoulos and Dvali [4]. Helioseismological and red-giant type considerations were performed by Cassisi et al. [14]. They calculated detailed solar models taking into account energy loss in Kaluza-Klein gravitons explicitly in their code and obtained the lower bound for M_s equal to $0.3 \text{ TeV}/c^2$. More stringent limit was derived from simulating the globular cluster Hertzsprung-Russel diagram [14] (implementing Kaluza-Klein graviton emissivity into FRANEC evolutionary code). By virtue of comparing predicted luminosity of RGB tip with observations Cassisi et al. obtained a "red-giant" bound for M_s to be $3 - 4 \text{ TeV}/c^2$. Our estimate of the string energy scale implied by recently reported stability of ZZ Ceti star G117-B15A

equal to $14.3 \text{ TeV}/c^2$ is much stronger than above mentioned stellar evolutionary bounds.

Among existing astrophysical bounds on Kaluza-Klein theories with large extra dimensions only the supernova constraints are more restrictive. They demand $M_s > 30\text{--}130 \text{ TeV}/c^2$ [11, 12] and are based on a different mechanism of Kaluza-Klein graviton emission – the nucleon-nucleon bremsstrahlung. On the other hand the lesson learned in testing the physics of axions shown that first straightforward supernova bounds were reduced by an order of magnitude when more accurate nuclear physics was employed [27]. Quite recently, the paper with improved calculations of nucleon-nucleon bremsstrahlung appeared [13] suggesting that former SN1987a bounds should be lowered by a factor of 0.65. It should also be noted that cosmological considerations [28] apparently provide much more stringent bounds than the supernova SN1987a.

The bound derived in this paper is based on white dwarf cooling. The physics underlying this process is very simple hence one can expect that the result is robust (within the framework of the theory proposed by Arkani-Hamed, Dimopoulos and Dvali [4]). Kaluza-Klein graviton emissivity at the relevant densities and temperatures of white dwarfs is dominated by the gravibremsstrahlung process taking place in the degenerate and isothermal core. The mass of the core (essentially equal to the mass of the star, since the outer helium layer comprises less than $0.01 M_{WD}$, its temperature and chemical composition can be reliably estimated by fitting evolutionary models to observational characteristics such like effective temperature or oscillation periods. The cooling rate of B117-G15A pulsating star is constrained by measurements which are performed with great accuracy. The most recent determination of the secular rate of change of the period [21] took into account all the periodicities and the error bars reported therein can be considered as safe. It can be argued [22] that remaining uncertainty – mostly from mode identification procedure and the precise physical characteristics (mass or temperature) is of order of $1 \times 10^{-11} \text{ s s}^{-1}$. Other effects like the contribution of the proper motion [26, 21] or the rate of reaction $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ which determines the stratification of the core at final stage of the asymptotic giant branch, both contribute an order of magnitude smaller value to the final uncertainty.

The detailed discussion of using a pulsational code coupled to evolutionary code aimed at constraining an axion mass by observed stability of the fundamental mode of G117-B15A can be found in a recent paper by Córscico et al. [22]. One of the conclusions formulated in [22] was that the bounds on axion mass derived from simple estimates like performed in the present paper (in a different context) or in [8] are in good agreement with evolutionary calculations. Although axion emissivity has different temperature dependence than that of Kaluza-Klein gravitons, one can expect the same for gravitons. Hence the B117-G15A pulsator remains an important tool for testing fundamental

physics.

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